

MATHEMATICS

Education must equip all students with mathematical skills and ways of thinking that provide them with the flexibility, adaptability, and creativity to function as productive citizens in the changing society of the twenty-first century. Mathematics understanding must extend beyond the skills of calculation and manipulation of numbers and symbols to the use of mathematics to investigate, predict, analyze, interpret, create, and evaluate.

Deep mathematical understanding develops over time. While performance indicators describe the knowledge and skills expected at a grade level, these concepts and skills may be introduced in previous years. They will also be used in later years as the foundations for more advanced topics or in new problem situations.

The use of “understand” in this document is intended to communicate the desired depth and breadth of mathematics programs for Maine students. To understand a procedure or concept means to be able to:

- communicate its meaning, its use, the results of its application, and its implications for a given context
- reason about it by making conjectures and justifying conclusions
- represent it in a variety of ways
- connect it to other ideas in and outside of mathematics, and
- know when and how to apply it to solve problems in mathematics and in other contexts.

Central to mathematical understanding is learning through problems that arise in mathematics and applied contexts. To this end students learn to identify problems, formulate approaches, carry out these approaches, and communicate and justify solutions. Mathematical reasoning pervades all areas of mathematics. Mathematical reasoning is manifested through classification, comparison, deduction, induction, generalization, justification, verification, and spatial visualization.

As growing mathematicians, students need to do mathematics and see themselves as capable of developing their own understanding of mathematical concepts, properties and procedures. Mathematics classrooms should provide practical experiences using mathematics in everyday applications and in other content areas, as well as explorations solely within mathematics. Discussing mathematics is an important component of developing mathematical understanding. Technology should be used as an aid to understanding mathematical ideas. Classrooms that reflect these beliefs prepare students to be confident and effective mathematical thinkers.

As lifelong learners students will research mathematics concepts and methods. They must learn about sources of mathematics information, how to read and comprehend mathematics, how to employ the mathematical ideas they learn, and how to communicate what they learn.

Maine should expect its students to enjoy, appreciate, and use mathematics. Students who are challenged to reach these goals and supported in reaching them will be better prepared for a future in which mathematics will be increasingly important in all areas of endeavor.

MATHEMATICS OUTLINE

A. Number

- Whole
- Rational
- Real

B. Data

- Measurement and Approximation
- Data Analysis
- Probability

C. Geometry

- Geometric Figures
- Geometric Measurement
- Transformations

D. Algebra

- Symbols and Expressions
- Equations and Inequalities
- Functions and Relations

DRAFT DOCUMENT -- PROPOSED REVISED MAINE LEARNING RESULTS STANDARDS -- DRAFT DOCUMENT

- A. **NUMBER:** Students use numbers in everyday and mathematical contexts to quantify or describe phenomena, develop concepts of operations with different types of numbers, use the structure and properties of numbers with operations to *solve* problems, and perform mathematical computations. Students develop number sense related to magnitude, estimation, and the effects of mathematical operations on different types of numbers. It is expected that students use numbers flexibly, using forms of numbers that best match a situation. Students compute efficiently and *accurately*. *Estimation* should always be used when computing with numbers or solving problems.

WHOLE NUMBER					
PK-2 PERFORMANCE INDICATORS					
<p>1 Students <i>understand</i> and use number notation and place value to 1000 in numerals.</p> <ul style="list-style-type: none"> a. Read and write numbers to 1000 using numerals. b. Recognize the place values of digits in numbers (hundreds, tens, and ones). c. Compare and order 1, 2, and 3-digit numbers. <p>2 Students <i>understand</i> and use procedures to add and subtract whole numbers with one and two digits.</p> <ul style="list-style-type: none"> a. Use and explain multiple strategies for computation. b. Use an operation appropriate to a given situation. 					
Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
<p>1 Students <i>understand</i> and use number notation and place value to 10,000 in numerals.</p> <ul style="list-style-type: none"> a. Read and write numbers up to 10,000 in numerals and words. b. Recognize the place values of 	<p>1 Students <i>understand</i> and use number notation and place value to 100,000.</p> <ul style="list-style-type: none"> a. Read and write numbers up to 100,000 in numerals and words. b. Recognize the place value of digits in numbers 	<p>1 Students <i>understand</i> and use number notation to 10 million in numerals and words.</p> <ul style="list-style-type: none"> a. Read and write numbers to 10 million in numerals. b. Round numbers to the place value 	<p>1 Students use factors and multiples.</p> <ul style="list-style-type: none"> a. Identify prime numbers and composite numbers and use their properties to <i>solve</i> problems. b. Use the property that every integer greater 	<p>No performance indicator.</p> <p>It is expected that students continue to use prior concepts and skills in new and familiar contexts.</p>	<p>No performance indicator.</p> <p>It is expected that students continue to use prior concepts and skills in new and familiar contexts.</p>

<p>digits in numbers up to 10,000.</p> <p>c. Compare and order numbers with up to 4 digits.</p> <p>2 Students <i>understand</i> and use procedures to add and subtract whole numbers with up to four digits.</p> <p>a. Display an understanding of the base ten place value system.</p> <p>b. Use an operation appropriate to a given situation.</p> <p>3 Students <i>understand</i> and apply meanings of multiplication and division.</p> <p>a. Multiply single-digit numbers and divide using single-digit divisors and up to two-digit</p>	<p>to 100,000.</p> <p>c. Compare and order numbers with up to 5 digits.</p> <p>d. Round numbers to the nearest 100 or 1000.</p> <p>2 Students <i>understand</i> and use the concepts of factor and multiple.</p> <p>a. Determine if a single-digit number is a factor of a given whole number.</p> <p>b. Determine if a whole number is a multiple of a given single-digit number.</p> <p>c. List the first 10 multiples of a given number.</p> <p>3 Students <i>understand</i> and use procedures to multiply and divide whole numbers by two-digit numbers.</p>	<p>appropriate for given contexts.</p> <p>c. Compare and order numbers up to 10 million.</p> <p>2 Students multiply and divide numbers up to four digits by numbers up to 2 digits, and by tens, hundreds, and thousands and <i>interpret</i> any remainders.</p> <p>3 Students <i>solve</i> problems requiring multiple operations – addition, subtraction, multiplication, and division and use the conventions of order of operations (no exponents expected).</p>	<p>than 1 can be written as a product of prime factors.</p> <p>c. <i>Interpret</i> and use exponential notation as repeated multiplication.</p> <p>d. Find the least common multiple and greatest common factor of two numbers.</p>		
--	---	---	---	--	--

<div>dividends.</div> <div>b. Use an operation appropriate to a given situation.</div> <div>c. Recognize and use models for multiplication and division situations.</div> <div>d. Use multiple strategies for multiplication and division.</div>	<div>a. Multiply up to four-digit numbers by a single-digit number.</div> <div>b. Multiply three-digit numbers by two-digit numbers.</div> <div>c. Divide whole numbers up to four digits by a single-digit number and by ten.</div>				
9-Diploma PERFORMANCE INDICATORS					
No performance indicator.					
It is expected that students continue to use prior concepts and skills in new and familiar contexts.					

RATIONAL NUMBER					
PK-2 PERFORMANCE INDICATORS					
3 Students recognize unit fractions including $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$.					
Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
<div>4 Students recognize, name, compare, illustrate, and use simple fractions.</div> <div>a. Recognize and</div>	<div>4 Students <i>understand</i>, name, compare, illustrate, combine, and use fractions.</div>	<div>4 Students <i>understand</i>, name, compare, illustrate, compute with, and use fractions.</div>	<div>2 Students express fractions greater than 0 as decimals, compare positive numbers that are written as fractions and decimals, and</div>	<div>1 Students use negative and positive rational numbers expressed as integers, fractions, and decimals.</div>	<div>1 Students express or <i>interpret</i> numbers using scientific notation from real-life contexts.</div>

<p>name fractions with denominators from 2-10.</p> <p>b. Recognize and name parts of a whole.</p> <p>c. Compare and order fractions with like numerators or with like denominators.</p>	<p>a. Add and subtract fractions with like denominators and use repeated addition to multiply a unit fraction by a whole number.</p> <p>b. List equivalent fractions.</p> <p>c. Represent fractions greater than one as mixed numbers and mixed numbers as fractions.</p> <p>5 Students <i>understand</i> and use number notation and place value in numbers with two decimal places in real world contexts including money.</p> <p>a. Compare, order, read, round and <i>interpret</i> decimals with up to two decimal places.</p> <p>b. Add and subtract</p>	<p>a. Add and subtract fractions with unlike denominators.</p> <p>b. Multiply a fraction by a whole number.</p> <p>5 Students <i>understand</i> and use number notation and place value in numbers with three decimal places.</p> <p>a. Compare, order, read, round, and <i>interpret</i> decimals with up to three decimal places.</p> <p>b. Add and subtract decimals with up to three decimal places.</p> <p>c. Multiply and divide decimals with up to three decimal places by a 2-digit whole number.</p> <p>d. Develop the concept of a fraction as division through</p>	<p>place them on the number line.</p> <p>3 Students add, subtract, multiply, and divide numbers expressed as fractions and as decimals including mixed numbers.</p> <p>4 Students <i>understand</i> how to express relative quantities as percentages, and as decimals and fractions.</p> <p>a. Use ratios to describe relationships between quantities.</p> <p>b. Use decimals, fractions, and percentages to express relative quantities.</p> <p>c. <i>Interpret</i> relative quantities expressed as decimals, fractions and percentages.</p>	<p>a. Recognize rational numbers as quotients of integers with a non-zero denominator and that rational numbers can be negative or positive.</p> <p>b. Compare signed rational numbers and place them on the number line.</p> <p>2 Students compute with signed rational numbers.</p> <p>a. Use and <i>interpret</i> exponents.</p> <p>b. Follow conventions of order of operations including exponents.</p> <p>3 Students <i>understand</i> that when the ratio of two varying quantities is constant,</p>	<p>a. Use positive and negative integer exponents for powers of ten.</p> <p>b. Convert between standard and scientific notation forms and compare the relative size of numbers including the <i>interpretation</i> of numbers as displayed on calculators and computers.</p>
---	--	--	--	---	--

	<p>decimals with up to two decimal places.</p> <p>c. Multiply and divide decimals with up to two decimal places by a 1- digit whole number.</p> <p>d. Connect equivalent decimals and fractions for $\frac{1}{10}$s, $\frac{1}{4}$s and $\frac{1}{2}$s in meaningful contexts.</p>	<p>expressing a fraction with denominators of 2, 4, 5, 10, as a decimal and the decimal as a fraction.</p> <p>6 Students <i>understand</i> concepts of positive and negative integers.</p> <p>a. Place positive and negative integers on a number line or scale.</p> <p>b. Compare and order positive and negative integers.</p> <p>c. Find the distance between two integers in a context.</p>	<p>5 Students multiply and divide decimals with up to 3-decimal places by tens, hundreds, and thousands.</p>	<p>the two quantities are in direct proportion.</p> <p>a. Use ratios to compare quantities and use comparison to solve problems.</p> <p>b. Identify proportional relationships.</p> <p>c. Use proportions to <i>solve</i> problems.</p> <p>4 Students <i>interpret</i> and use percents to <i>solve</i> problems.</p> <p>a. Use percents when comparing fractional parts of sets of unequal size.</p> <p>b. <i>Solve</i> practical problems involving percents.</p>	
--	---	---	--	---	--

9-Diploma PERFORMANCE INDICATORS

No performance indicator.

Students continue to use prior concepts and skills in new and familiar contexts.

REAL NUMBER					
PK-2 PERFORMANCE INDICATORS					
No performance indicator.					
Students are expected to use only rational numbers at this level.					
Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
<p>No performance indicator.</p> <p>Students are expected to use only rational numbers at this level.</p>	<p>No performance indicator.</p> <p>Students are expected to use only rational numbers at this level.</p>	<p>No performance indicator.</p> <p>Students are expected to use only rational numbers at this level.</p>	<p>No performance indicator.</p> <p>Students use rational numbers including rational approximations for pi or square roots.</p>	<p>No performance indicator.</p> <p>Students use rational numbers including rational approximations for pi or square roots.</p>	<p>1 Students <i>understand</i> the set of real numbers as containing the rational numbers and the irrational numbers.</p> <ul style="list-style-type: none"> a. Know that there are real numbers that are not rational numbers. b. Know some common examples of irrational numbers such as π or those arising from square roots. c. Use square roots. Be able to <i>estimate</i> the value of the square roots of

					whole numbers and place them on the number line.
--	--	--	--	--	--

9-Diploma PERFORMANCE INDICATORS

1 Students know how to represent and use real numbers.

- a. Use the concept of n^{th} root.
- b. *Estimate* the value of roots and use technology to approximate them.
- c. Compute using laws of exponents.
- d. Multiply and divide numbers expressed in scientific notation.
- e. *Understand* that some quadratic equations do not have real solutions and that the set of real numbers can be extended to allow for solutions to these equations.

B. DATA: Students make measurements and collect, display, evaluate, analyze, and compute with data to describe or *model* phenomena and to make decisions based on data. Students compute statistics to summarize data sets and use concepts of probability to make predictions and describe the uncertainty inherent in data collection and measurement. It is expected that when working with measurements students:

- *Understand* that most measurements are approximations and that taking repeated measurements reveals this variability.
- *Understand* that a number without a *unit* is not a measurement. Thus an appropriate *unit* must always be attached to a number to provide a measurement.
- *Understand* that the *precision* and *accuracy* of a measurement depends on selecting the appropriate tools and *units*.
- Use *estimation* comparing measures to *benchmarks* appropriate to the type of measure and *units*.

MEASUREMENT AND APPROXIMATION

PK-2 PERFORMANCE INDICATORS

1 Students *understand* and use *units* of time, temperature, and money.

- a. Apply and use sequences of hours in a day, days in a week, and months in a year.
- b. Tell time to the hour and half hour.
- c. Identify and give the value of different coins.
- d. Find the total value of collections of coins up to \$1.00.
- e. Read temperature on thermometers with scales marked with one degree intervals.

Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
<p>1 Students <i>understand</i> and use measurement of time and temperature.</p> <p>a. Select appropriate tools and <i>units</i>.</p> <p>b. <i>Solve</i> and <i>justify</i> problems with these measures.</p>	<p>1 Students <i>understand</i> and use measurement of time, capacity, and temperature.</p> <p>a. Select appropriate tools and <i>units</i> for these measures.</p> <p>b. <i>Solve</i> and <i>justify</i> problems with these measures.</p>	<p>1 Students <i>understand</i> and use measures of elapsed time, temperature, capacity, mass, and weight.</p> <p>a. Select appropriate tools and <i>units</i> mass in grams, weight in pounds.</p> <p>b. <i>Solve</i> and <i>justify</i> problems with these measures.</p>	<p>1 Students convert within measurement systems.</p> <p>a. <i>Solve</i> problems where different <i>units</i> are used within the metric and traditional systems of measurement.</p>	<p>No performance indicator.</p> <p>Although no performance indicators are stated at this level, it is expected that students continue to use prior concepts and skills in new and familiar concepts.</p>	<p>1 Students <i>understand</i> and use <i>derived measures</i> (measurements expressed as rates).</p> <p>a. Calculate measures using multiple attributes including speed (distance per time).</p> <p>b. <i>Solve</i> for an unknown component of a measure including finding time given average speed and distance.</p> <p>2 Students convert across measurement systems and within a system for different <i>units</i> in <i>derived measures</i>.</p> <p>a. Approximate metric and</p>

					<p>customary equivalents given a conversion factor.</p> <p>b. Convert <i>derived measures</i>, including feet per second to miles per hour.</p>
--	--	--	--	--	---

9-Diploma PERFORMANCE INDICATORS

1 Students *understand* the relationship between *precision* and *accuracy*.

- a. Express answers to a reasonable degree of *precision* in the context of a given problem.
- b. Represent an approximate measurement using appropriate numbers of significant figures.
- c. Know that most measurements are approximations and explain why it is useful to take the mean of repeated measurements.

DATA ANALYSIS

PK-2 PERFORMANCE INDICATORS

2 Students read, construct, and *interpret* picture graphs.

Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
2 Students read, construct and <i>interpret</i> bar graphs.	2 Students collect and represent data in tables, line plots, and bar graphs, and read and <i>interpret</i> these types of data displays.	<p>2 Students read, construct and <i>interpret</i> line graphs.</p> <p>3 Students find and use median, mode, and range for a set of data.</p>	<p>2 Students read and <i>interpret</i> pie charts.</p> <p>3 Students find and compare the mean, median, mode, and range for sets of data.</p>	<p>1 Students use graphs and charts to represent, organize, <i>interpret</i>, and draw inferences from data.</p> <p>a. <i>Create</i> tables, pictograms, bar</p>	3 Students use the mean, median, mode, range, and quartiles to <i>solve</i> problems involving raw data and information from data displays.

				<p>graphs, line graphs, pie charts, stem and leaf plots, box and whiskers plots, and histograms using pencil and paper and electronic technologies.</p> <p>b. Draw conclusions based on graphs and charts including tables, pictograms, bar graphs, line graphs, pie charts, stem and leaf plots, box and whiskers plots, and histograms.</p>	
--	--	--	--	---	--

9-Diploma PERFORMANCE INDICATORS

2 Students *understand* correlation and cause and effect.

- a. Recognize when correlation has been confused with cause and effect.
- b. *Create* and *interpret* scatter plots and *estimate* correlation and lines of best fit.
- c. Recognize positive and negative correlations based on data from a table or scatter plot.
- d. *Estimate* the strength of correlation base upon a scatter plot.

3 Students *understand* and know how to describe distributions and find and use descriptive statistics for a set of data.

- a. Find and apply range, quartiles, mean absolute deviation, and standard deviation (with technology) of a set of data.

- b. *Interpret*, give examples of and describe key differences between different types of distributions: uniform, normal and skewed.
- c. For the sample mean of normal distributions, use the standard deviation for a group of observations to establish 90%, 95%, or 99% confidence intervals.

4 Students *understand* that the purpose of random sampling is to reduce bias when creating a representative sample for a set of data.

- a. Describe and account for the difference between sample statistics and statistics describing the distribution of the entire population.
- b. Recognize that sample statistics produce *estimates* for the distribution of an entire population, and recognize that larger sample sizes will produce more reliable *estimates*.
- c. Apply methods of *creating* random samples and recognize possible sources of bias in samples.

PROBABILITY

PK-2 PERFORMANCE INDICATORS

No performance indicator.

While students are expected to have experiences with probability in these grades, it is not expected that the knowledge be secure.

Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
<p>No performance indicator.</p> <p>While students are expected to have experiences with probability in grade 3, it is not expected that the knowledge be secure.</p>	<p>No performance indicator.</p> <p>While students are expected to have experiences with probability in grade 4, it is not expected that the knowledge be secure.</p>	<p>No performance indicator.</p> <p>While students are expected to have experiences with probability in grade 5, it is not expected that the knowledge be secure.</p>	<p>No performance indicator.</p> <p>While students are expected to have experiences with probability in grade 6, it is not expected that the knowledge be secure.</p>	<p>2 Students <i>understand</i> and apply concepts of probability to simple events.</p> <ul style="list-style-type: none"> a. Describe events as likely or unlikely and discuss the concept of likelihood using such words as certain, equally 	<p>4 Students <i>understand</i> and apply concepts of probability.</p> <ul style="list-style-type: none"> a. Use appropriate terminology to describe complementary and mutually exclusive events. b. Use an <i>understanding</i> of relative

				<p>likely, and impossible.</p> <p>b. Predict the probability of outcomes of simple experiments and verify predictions using the <i>understanding</i> that the probability of an occurrence is the ratio of the number of actual occurrences to the number of possible occurrences.</p> <p>c. <i>Interpret</i> probabilities between and including zero and one and explain why zero and one are the upper and lower limits for probability values.</p>	<p>frequency to make and test conjectures about results of experiments and simulations.</p> <p>c. Compute probabilities for compound events, using such methods as organized lists, tree diagrams, and area models.</p>
9-Diploma PERFORMANCE INDICATORS					
<p>5 Students <i>understand</i> the relationship of probability to relative frequency and know how to find the probability of compound events.</p> <p>a. Find the expected frequency of an event.</p>					

- b. Find the expected value of events.
- c. Find the probability of compound events including independent and dependent events.

- C. **GEOMETRY:** Students use measurement and observation to describe objects based on their sizes and shapes, *model* or construct two- and three-dimensional objects, *solve* problems involving geometric properties, compute areas and volumes based on object properties and dimensions, and perform transformations on geometric figures. When making or calculating measures, students use *estimation* to check the reasonableness of results.

GEOMETRIC FIGURES					
PK-2 PERFORMANCE INDICATORS					
<p>1 Students recognize, <i>classify</i>, and <i>create</i> geometric figures in two and three dimensions.</p> <ul style="list-style-type: none"> a. Identify shapes in the physical environment. b. <i>Classify</i> figures as circles, triangles, and quadrilaterals by focusing on their properties. c. <i>Create</i> shapes by using objects to combine and <i>decompose</i> other shapes. 					
Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
<p>1 Students identify, describe, and <i>classify</i> familiar two-dimensional shapes.</p> <ul style="list-style-type: none"> a. Describe and <i>classify</i> two-dimensional shapes according to the number of vertices and by number, length and shape of sides. b. Know how to put 	<p>1 Students identify and name angles, lines, relationships between lines, quadrilaterals, and triangles.</p> <ul style="list-style-type: none"> a. Identify perpendicular and parallel lines and sides. b. Identify and sketch the following figures: rectangle, square, parallelogram, 	<p>1 Students identify, describe and <i>classify</i> solid figures.</p> <ul style="list-style-type: none"> a. Identify edges, vertices and faces in three-dimensional figures. b. Describe and <i>classify</i> solid figures according to the number of edges, faces, and vertices as well as the 	<p>1 Students represent solid figures in two dimensions.</p> <ul style="list-style-type: none"> a. Represent cubes, prisms, and square- or triangular-based pyramids using <i>nets</i>. b. Recognize and <i>classify</i> solids presented in picture views. c. Sketch 3-D figures. 	<p>1 Students <i>understand</i> angle properties of lines in the plane.</p> <ul style="list-style-type: none"> a. Identify and name straight angles, angles at a point, and vertical angles and use these measures to find the measures of unknown angles. b. Recognize that the measures 	<p>1 Students know and use properties of polygons.</p> <ul style="list-style-type: none"> a. Use the triangle inequality. b. Find the sum of the measures of the interior angles of a polygon. c. Use the property that the sum of the measures of the exterior angles of a

<p>shapes together and take them apart to form other shapes.</p> <p>c. Identify edges, vertices, and right angles in two-dimensional shapes.</p> <p>d. Tell whether a given angle is greater or smaller than a right angle.</p>	<p>rhombus, and trapezoid.</p> <p>c. Identify and sketch the following triangles: isosceles, equilateral, acute, obtuse, and right.</p>	<p>shapes of faces.</p>		<p>that form straight angles add to 180° and the measures of angles at a point add to 360°.</p> <p>c. Recognize that vertical angles are equal.</p>	<p>polygon is 360 degrees.</p> <p>2 Students know and use angle properties of parallel lines to <i>solve</i> problems and determine geometric relationships.</p> <p>a. Know and use properties of angles created when parallel lines are cut by a transversal.</p> <p>b. Use angle properties to determine whether lines are parallel.</p> <p>c. Know and use properties of angles created by parallel lines to determine the angle properties of trapezoids and parallelograms and apply these properties in problem situations.</p>
---	---	-------------------------	--	---	---

					3 Students know and use the Pythagorean Theorem.
--	--	--	--	--	--

9-Diploma PERFORMANCE INDICATORS

1 Students *justify* statements about polygons and *solve* problems.

- a. Use the properties of triangles to prove theorems about figures and relationships among figures.
- b. *Solve* for missing dimensions based on congruence and similarity.
- c. Use the Pythagorean Theorem in situations where right triangles are created by adding segments.
- d. Use the distance formula.

2 Students *justify* statements about circles and *solve* problems.

- a. Use the concepts of central and inscribed angles to *solve* problems and *justify* statements.
- b. Use the relationships among arc length, circumference, and area of circles, and sectors to *solve* problems and *justify* statements.

3 Students *understand* and use basic ideas of trigonometry.

- a. Identify and find the value of trigonometric ratios for angles in right triangles.
- b. Use trigonometry to *solve* for missing lengths in right triangles.
- c. Use inverse trigonometric functions to find missing angles in right triangles.

GEOMETRIC MEASUREMENT

PK-2 PERFORMANCE INDICATORS

2 Students *understand* how to measure length and capacity and use appropriate *units*.

- a. Measure length and capacity by *direct and indirect comparison*.
- b. Measure the length and capacity of objects using non-standard *units*.
- c. Measure the length of objects to whole inches and centimeters.

Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
<p>2 Students <i>understand</i> how to find the distance around a figure.</p> <p>a. Calculate and measure the distance around a figure whose perimeter is comprised of straight edges.</p>	<p>2 Students <i>understand</i> the concept of area of a figure.</p> <p>a. Find the area of shapes in non-standard <i>units</i> (e.g., <i>estimate</i> the number of whole square <i>units</i> that cover a figure).</p> <p>b. Find the area of squares and other rectangles in standard <i>units</i>.</p> <p>c. Recognize and <i>estimate</i> the relative sizes of 1 square meter and 1 square centimeter and 1 square inch and 1 square foot.</p>	<p>2 Students find the area of triangles and quadrilaterals.</p> <p>a. Know how to derive and use the formula, $A = (1/2) bh$ for the area of a triangle.</p> <p>b. Find the area of parallelograms.</p> <p>3 Students <i>understand</i> how to find the volume and surface area of rectangular prisms.</p> <p>a. Know how to build solids with unit cubes and find their volume.</p> <p>b. Recognize and <i>estimate</i> the relative sizes of 1 cubic meter and 1 cubic centimeter. Or 1 cubic inch and 1 cubic foot.</p>	<p>2 Students find the perimeters and areas of geometric figures.</p> <p>a. Triangles b. Quadrilaterals c. Circles</p> <p>3 Students find the volume and surface areas of right prisms with bases that are triangles and quadrilaterals.</p>	<p>2 Students <i>solve</i> problems involving perimeter and area.</p> <p>a. <i>Solve</i> problems involving the area and perimeter of regions in the plane bounded by line segments and circular arcs.</p> <p>b. <i>Solve</i> problems involving the area of combined figures.</p>	<p>2 Students find the volume and surface area of prisms, pyramids, cylinders, and other figures <i>composed</i> of these solids.</p> <p>a. Apply the <i>understanding</i> that the volume of prisms and cylinders can be found by multiplying the area of a base by the height of the solid.</p> <p>b. Apply the <i>understanding</i> that the volume of pyramids can be found by multiplying the area of a base by 1/3 the height of the solid.</p>

		<ul style="list-style-type: none">c. Know how to derive and use the formula (length x width x height) for the volume of a rectangular prism.d. <i>Create nets</i> to aid visualization and computation. <p>4 Students <i>understand</i> how to describe position and direction in two dimensions.</p> <ul style="list-style-type: none">a. Locate points on the Cartesian plane.b. Determine horizontal and vertical distance on the coordinate plane.c. Measure angles in degrees.			
--	--	--	--	--	--

9-Diploma PERFORMANCE INDICATORS

4 Students find the surface area and volume of 3-D objects.

- a. Find the volume and surface area of cones and spheres.
- b. Use formulas to determine the effect of changes in linear dimensions on the volume and surface area of similar 3-D figures.

TRANSFORMATIONS

PK-2 PERFORMANCE INDICATORS

No performance indicator.

While students are expected to have experiences with symmetry, transformations, and congruence in these grades, it is not expected that the knowledge be secure.

Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
<p>No performance indicator.</p> <p>While students are expected to have experiences with symmetry, transformations and congruency in grade 3 it is not expected that the knowledge be secure.</p>	<p>3 Students recognize congruent figures and line symmetry in figures.</p> <ul style="list-style-type: none"> a. Recognize whether a line is a line of symmetry in a figure. b. Complete a symmetric figure given a line of symmetry. c. Recognize 	<p>5 Students reflect, slide, and rotate plane figures.</p> <ul style="list-style-type: none"> a. Identify figures with rotational or line symmetry. b. <i>Create</i> figures with rotational or line symmetry c. Slide, rotate or reflect figures to create patterns or demonstrate congruence. 	<p>4 Students <i>understand</i> and use reflections, rotations, and translations to define and identify congruent plane figures.</p> <ul style="list-style-type: none"> a. Apply the <i>understanding</i> that if a plane figure can be laid on top of another plane figure by rotations, 	<p>3 Students <i>understand</i> and use the concept of scale drawings to enlarge or reduce two-dimensional plane figures.</p> <ul style="list-style-type: none"> a. Use the concept of scale factors when enlarging or reducing and recognize the invariance of shape. 	<p>No performance indicator.</p> <p>It is expected that students continue to use prior concepts and skills in new and familiar contexts.</p>

	congruent figures.		translations, or reflections then the figures are congruent. 5 Students <i>understand</i> how to use proportional relationships to make indirect linear measurements and use scale drawings to make linear measurements.	b. Apply the <i>understanding</i> that enlargement or reduction by a scale factor leaves angle measures unchanged. c. Identify similar figures and name corresponding parts.	
--	--------------------	--	---	---	--

9-Diploma PERFORMANCE INDICATORS

No performance indicator.

It is expected that students continue to use prior concepts and skills in new and familiar contexts. Methods of transformational geometry might also be used in Geometric Figures 9-Diploma performance indicator 1.

- D. **ALGEBRA:** Students use symbols to represent or *model* quantities, patterns, and relationships and use symbolic manipulation to *evaluate* expressions and *solve* equations. Students *solve* problems using symbols, tables, graphs, and verbal rules choosing the most effective representation and converting among representations.

SYMBOLS AND EXPRESSIONS

PK-2 PERFORMANCE INDICATORS

1 Students *understand* how to represent quantities as simple expressions using addition and subtraction.

- Show that any quantity can be represented by equivalent expressions e.g., $4 + 5 + 1$; $3 + 3 + 3 + 1$; $9 + 1$ each represents the quantity 10.
- Know that addition is commutative and apply this *understanding* in computation and problem-solving.
- Know that addition and subtraction are inverse operations and apply this *understanding* in computation and problem-solving.

Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
1 Students use equivalent expressions to aid computation such as knowing that $43 + 56$ is the same as $40 + 3 + 50 + 6$.	1 Students <i>create</i> and <i>evaluate</i> simple expressions in the context of numbers and operations as described in Standard A: Number for this grade level. a. <i>Create</i> and <i>evaluate</i> expressions with no more than 2 variables.	1 Students <i>create</i> and <i>evaluate</i> simple expressions in the context of numbers and operations as described in Standard A: Number for this grade level. a. <i>Create</i> and <i>evaluate</i> expressions with no more than 3 variables.	1 Students <i>create</i> and <i>evaluate</i> expressions. a. <i>Create</i> and <i>evaluate</i> expressions using whole numbers. b. <i>Create</i> and <i>evaluate</i> expressions using positive fractions including decimals.	1 Students <i>create</i> and <i>evaluate</i> expressions. a. <i>Create</i> and <i>evaluate</i> expressions using integers. b. <i>Create</i> and <i>evaluate</i> expressions using rational numbers.	1 Students <i>create</i> , <i>evaluate</i> and manipulate expressions. a. Add and subtract linear expressions. b. Apply the properties of the real number system (e.g., distributive and associative laws) to create equivalent expressions.
9-Diploma PERFORMANCE INDICATORS					
1 Students <i>understand</i> and use polynomials, and expressions with rational exponents. a. <i>Simplify</i> expressions with rational exponents. b. Add, subtract, and multiply polynomials. c. Factor the common term out of polynomial expressions. d. Divide polynomials by $(ax+b)$.					

EQUATIONS AND INEQUALITIES					
PK-2 PERFORMANCE INDICATORS					
<p>2 Students <i>understand</i> that the equal sign means, "is the same as."</p> <p>a. Identify true and false number sentences.</p> <p>b. Describe what makes number sentences true or false and apply this knowledge.</p> <p>c. Find solutions for unknowns in simple open number sentences such as $12 = 4 + []$.</p>					
Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
<p>2 Students find the unknown in simple equations (or open sentences) in the context of numbers and operations as described in Standard A: Number for this grade level such as:</p> <p>$3 + 5 = [] + 3$ $3 + 9 = [] + 10$ $[] + () = 10$.</p>	<p>2 Students find the unknown in simple equations in the context of numbers and operations as described in Standard A: Number for this grade level such as:</p> <p>$3 \cdot b = 12$ $3 + 4 = x + 5$ $6 \times 5 = 3 \times []$.</p>	<p>2 Students find the unknown in simple equations in the context of numbers and operations as described in Standard A: Number for this grade level such as:</p> <p>$39 - k = 39 - 40$ $78 + b = 57 + 79$ $30 \times A = 276$ $(3 + 4) \times 6 = 6 \times []$ $3 \times 15 = 3 \times (10 + [])$.</p>	<p>2 Students recognize and <i>solve</i> problems involving linear equations and recognize examples and non-examples of linear equations.</p> <p>a. <i>Solve</i> equations of the form $ax \pm b = c$ where a, b, and c are whole numbers.</p> <p>b. Recognize from a table whether a relationship has a constant rate of change.</p>	<p>2 Students <i>understand</i> and <i>solve</i> problems involving linear equations and know that a linear equation can be written in the form $0 = ax + b$.</p> <p>a. <i>Solve</i> equations of the form $ax + b = c$ where a, b, and c are positive rational numbers or positive or negative integers.</p> <p>b. Convert equations to $0 = ax + b$ form.</p>	<p>2 Students <i>understand</i> and <i>solve</i> problems involving linear equations.</p> <p>a. Be able to <i>solve</i> any linear equation including linear equations of the form $ax + b = cx + d$.</p> <p>b. Recognize that, in general, linear equations have just one solution—but know also that some linear equations can have no solution and those linear equations that are identities</p>

					<p>have every value of x as a solution.</p> <p>c. Use graphs to <i>estimate</i> solutions to equations and systems of equations, check algebraic approaches, provide alternative solution paths, and communicate the solution to a problem.</p> <p>3 Students <i>understand</i> and <i>solve</i> linear inequalities in one unknown.</p> <p>a. Represent problem situations as inequalities.</p> <p>b. <i>Solve</i> linear inequalities.</p> <p>c. <i>Interpret</i> the solutions to linear inequalities.</p>
--	--	--	--	--	--

9-Diploma PERFORMANCE INDICATORS

2 Students *solve* families of equations and inequalities.

- a. *Solve* systems of linear equations and inequalities in two unknowns and interpret their graphs.
- b. *Solve* quadratic equations: graphically, by factoring in cases where factoring is efficient, and by applying the quadratic formula.
- c. *Solve* simple rational equations similar to $\frac{1}{2x+1} = 5$.
- d. *Solve* absolute value equations and inequalities and interpret the results.
- e. Apply the *understanding* that the solution(s) to equations of the form $f(x) = g(x)$ are the x -value(s) of the point(s) of intersection of the graphs of $f(x)$ and $g(x)$ and common outputs in table of values.
- f. Explain why the coordinates of the point of intersection of the lines represented by a system of equations is its solution and apply this *understanding* to solving problems.

3 Students *understand* and apply ideas of logarithms.

- a. Use and *interpret* logarithmic scales.
- b. *Solve* equations in the form of $x = b^y$ using the equivalent form $y = \log_b x$.

FUNCTIONS AND RELATIONS

PK-2 PERFORMANCE INDICATORS

3 Students *understand* how to *create*, identify, describe, and extend patterns given a pattern or a rule.

- a. Describe, extend, and *create* a repeating pattern.
- b. Describe, extend, and *create* growing patterns.

Grade 3 PERFORMANCE INDICATORS	Grade 4 PERFORMANCE INDICATORS	Grade 5 PERFORMANCE INDICATORS	Grade 6 PERFORMANCE INDICATORS	Grade 7 PERFORMANCE INDICATORS	Grade 8 PERFORMANCE INDICATORS
<p>3 Students <i>understand</i> arithmetic relationships among positive whole numbers.</p> <p>a. Use the inverse relationships between addition and subtraction and between multiplication and division and the commutative laws of multiplication and addition to <i>solve</i> problems.</p> <p>b. Be able to show that for whole numbers subtraction and division are not commutative and show that multiplication and addition are commutative.</p> <p>4 Students <i>create</i>, describe, explain and</p>	<p>3 Students use tables, rules, diagrams, and patterns to represent the relationship between quantities and to extend sequences.</p>	<p>3 Students use tables, rules, diagrams, and graphs to represent and analyze the relationship between quantities.</p>	<p>3 Students use tables, formulas, diagrams, and graphs to analyze relationships between quantities.</p> <p>a. Use tables, formulas and graphs to analyze constant difference (additive) relationships.</p> <p>b. Use tables, formulas, and graphs to analyze constant ratio (multiplicative) relationships.</p>	<p>3 Students <i>understand</i> and use directly proportional relationships, $y = kx$.</p> <p>a. Recognize directly proportional relationships by information in a table, graph, or formula.</p> <p>b. Translate common directly proportional relationships into symbolic statements and graphs.</p> <p>c. <i>Interpret</i> the slope and y-intercept of the graph of $y = kx$ in terms of a given context.</p>	<p>4 Students <i>understand</i> and use the basic properties of linear relationships, $y = kx + b$.</p> <p>a. <i>Understand</i> that linear relationships are characterized by a constant rate of change, k.</p> <p>b. <i>Understand</i> that the graph of a linear relationship $y = kx + b$ is a line where the slope is k and b is the y-coordinate of the point where the graph crosses the y-axis (i.e., value of y when $x = 0$).</p> <p>c. Translate common linear phenomena into symbolic statements and graphs and</p>

extend patterns with numbers and geometric objects.					interpret the slope and y-intercept of the graph of $y = kx + b$ in terms of the original situation.
---	--	--	--	--	--

9-Diploma PERFORMANCE INDICATORS

4 Students *understand* and *interpret* the characteristics of functions using graphs, tables, and algebraic techniques.

a. Recognize the graphs and sketch graphs of the basic functions

$$f(x) = x^n, \text{ where } n = 1 \text{ to } 3;$$

$$f(x) = ax^2 + bx + c;$$

$$f(x) = \sqrt{x};$$

$$f(x) = |x|;$$

$$f(x) = \frac{a}{x};$$

$$f(x) = a^x; \text{ and}$$

$$f(x) = kx + b.$$

b. Use concepts such as domain, range, zeros, intercepts, and maximum and minimum values.

c. Use the concepts of average rate of change (table of values) and increasing and decreasing over intervals and use these characteristics to compare functions.

5 Students express relationships *recursively* and use *iterative* methods to *solve* problems.

a. Express the (n+1)st term in terms of the nth term and describe relationships in terms of a starting point and rule followed to transform one term to the next.

b. Use technology to perform repeated calculations to develop solutions to real life problems involving linear, exponential, and other patterns of change.